# 5: Dividing Whole Numbers by Fractions

Question: What is	the value of $3 \div \frac{1}{4}$	
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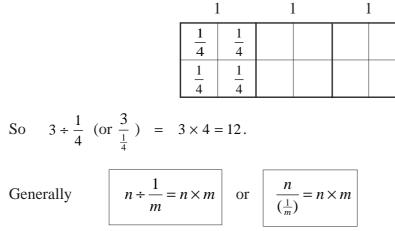
Misconception	Correct				
The value of $3 \div \frac{1}{4}$ is equivalent to $3 \div 4$	The division $3 \div \frac{1}{4}$ means how many $\frac{1}{4}$				
and hence has value $\frac{3}{4}$ or 0.75	are there in the number 3. Clearly there are				
	4 quarters in 1 and hence $3 \times 4$ (=12) in 3.				
Similarly $5 \div \frac{1}{2}$ is equivalent to $5 \div 2$ and	So $3 \div \frac{1}{4} = 12$				
hence has value 2.5 or $2\frac{1}{2}$					

## **Further Explanation**

We must learn NEVER to be influenced by what things look like: the meaning of dividing by 2, dividing by 5, etc. is clear: the concept of dividing by a quarter is, however, less straightforward and requires more thought.

Think of  $3 \div \frac{1}{4}$  as 'the number of  $\frac{1}{4}$ 's that fit into 3'.

There are 4 quarters in 1, so in 3 there are  $3 \times 4$  quarters in 3 as can be seen in the diagram below.



Hence, for example,  $5 \div \frac{1}{2} = 5 \times 2 = 10$ .

#### **Misconception 5**

There is another way to approach this task *logically* which we will demonstrate with  $6 \div \frac{3}{5}$ .

Use the problem solving method – 'if you are having difficulties, find something similar which you know you CAN do and work out the difference between this and the problem given'.

The difficult part here is *dividing by a fraction*.

Start with something similar which is straightforward: just divide the 6 by 3  $\left(\frac{6}{3}\right)$ . Now

continue by examining the effect of the difference between what we did and what was given (using clearer terminology to refer to division, i.e. divide *between*).

We divided the 6 by 3 instead of by the given  $\frac{3}{5}$  (which is, of course, less than 3).

When a cake is divided between a certain number of people, each gets a certain portion. Dividing it between *fewer people* results in each one receiving a larger portion. How much larger? If it is divided between, say, 5 times fewer people, each portions would become 5 times larger.

We arrived at 2 by dividing the 6 by 3. We *should* have divided by something that is 5 times smaller than the 3, (by  $\frac{3}{5}$ ), so, the result should be 5 times *larger* than the  $\frac{6}{3}$ . Thus we deduce that our  $6 \div \frac{3}{5}$  must mean  $\frac{6}{3} \times 5$  (= 10). Generalising,

$$a \div \frac{b}{c} \left( \text{or } \frac{a}{\frac{b}{c}} \right) = \frac{a}{b} \times c \left( = \frac{a \times c}{b} = a \times \frac{c}{b} \right)$$

Yet another way of determining  $\frac{3}{\frac{1}{4}}$  is to forget about the unclear meaning of *dividing by a fraction* and to do whatever yields a result which doesn't contradict other things that are already established.

Whatever we mean by  $\frac{p}{k}$ , we already know that its result, *r*, must be such that  $r \times k$  will be equal to *p*. i.e. in  $\frac{p}{k} = r$ , *r* must be such that  $r \times k = p$ , (e.g.  $\frac{187}{11}$  is 17 because  $17 \times 11 = 187$ ). Following this for  $\frac{3}{\frac{1}{4}}$ , we simply seek a result which gives 3 when multiplied by a  $\frac{1}{4}$ . The question then becomes: "what times a quarter is 3?", or using a familiar rephrasing "a quarter of what is 3?" (The answer is of course 12.) In summary

to determine the value of r in  $\frac{3}{\frac{1}{4}} = r$ , find which value of r satisfies  $r \times \frac{1}{4} = 3$ 

## **Follow-up Exercises**

1. Calculate the value of:

(a) 
$$4 \div \frac{1}{2}$$
 (b)  $3 \div \frac{1}{3}$  (c)  $6 \div \frac{1}{4}$  (d)  $10 \div \frac{1}{5}$ 

- (e)  $4 \div \frac{1}{3}$  (f)  $5 \div \frac{1}{4}$  (g)  $20 \div \frac{1}{5}$  (h)  $6 \div \frac{1}{6}$
- 2. Calculate the value of:

(a) 
$$4 \div \frac{2}{5}$$
 (b)  $3 \div \frac{3}{4}$  (c)  $10 \div \frac{2}{3}$  (d)  $4 \div \frac{3}{4}$ 

(e) 
$$6 \div \frac{3}{5}$$
 (f)  $1 \div \frac{2}{5}$  (g)  $7 \div \frac{5}{8}$  (h)  $20 \div \frac{4}{5}$ 

### Answers

1.	(a) 8	(b) 9	(c) 24	(d) 50	(e) 12	(f) 20	(g) 100	(h) 36
2.	(a) 10	(b) 4	(c) 15	(d) $\frac{16}{3}$	(e) 10	(f) $\frac{5}{2}$	(g) $\frac{56}{5}$	(h) 25