# 5: Dividing Whole Numbers by Fractions

*Question*: **What is the value of $3 \div \dfrac{1}{4}$ ?**

| Misconception | Correct |
|---|---|
| The value of $3 \div \dfrac{1}{4}$ is equivalent to $3 \div 4$ and hence has value $\dfrac{3}{4}$ or 0.75<br><br>Similarly $5 \div \dfrac{1}{2}$ is equivalent to $5 \div 2$ and hence has value 2.5 or $2\dfrac{1}{2}$ | The division $3 \div \dfrac{1}{4}$ means how many $\dfrac{1}{4}$ are there in the number 3. Clearly there are 4 quarters in 1 and hence $3 \times 4\,(=12)$ in 3. So $3 \div \dfrac{1}{4} = 12$ |

## Further Explanation

We must learn NEVER to be influenced by what things look like: the meaning of dividing by 2, dividing by 5, etc. is clear: the concept of dividing by a quarter is, however, less straightforward and requires more thought.

Think of $3 \div \dfrac{1}{4}$ as 'the number of $\dfrac{1}{4}$'s that fit into 3'.

There are 4 quarters in 1, so in 3 there are $3 \times 4$ quarters in 3 as can be seen in the diagram below.



So $\quad 3 \div \dfrac{1}{4}$ (or $\dfrac{3}{\frac{1}{4}}$) $\quad = \quad 3 \times 4 = 12$.

Generally $\qquad \boxed{n \div \dfrac{1}{m} = n \times m}$ or $\boxed{\dfrac{n}{\left(\frac{1}{m}\right)} = n \times m}$

Hence, for example, $5 \div \dfrac{1}{2} = 5 \times 2 = 10$.

## Misconception 5

There is another way to approach this task *logically* which we will demonstrate with $6 \div \dfrac{3}{5}$.

Use the problem solving method – 'if you are having difficulties, find something similar which you know you CAN do and work out the difference between this and the problem given'.

The difficult part here is *dividing by a fraction*.

Start with something similar which is straightforward: just divide the 6 by 3 $\left( \dfrac{6}{3} \right)$. Now continue by examining the effect of the difference between what we did and what was given (using clearer terminology to refer to division, i.e. divide *between*).

We divided the 6 by 3 instead of by the given $\dfrac{3}{5}$ (which is, of course, less than 3).

When a cake is divided between a certain number of people, each gets a certain portion. Dividing it between *fewer people* results in each one receiving a larger portion. How much larger? If it is divided between, say, 5 times fewer people, each portions would become 5 times larger.

We arrived at 2 by dividing the 6 by 3. We *should* have divided by something that is 5 times smaller than the 3, (by $\dfrac{3}{5}$), so, the result should be 5 times *larger* than the $\dfrac{6}{3}$. Thus we deduce that our $6 \div \dfrac{3}{5}$ must mean $\dfrac{6}{3} \times 5$ ( = 10 ). Generalising,

$$ a \div \frac{b}{c} \left( \text{or } \frac{a}{\frac{b}{c}} \right) = \frac{a}{b} \times c \left( = \frac{a \times c}{b} = a \times \frac{c}{b} \right) $$

Yet another way of determining $\dfrac{3}{\frac{1}{4}}$ is to forget about the unclear meaning of *dividing by a fraction* and to do whatever yields a result which doesn't contradict other things that are already established.

Whatever we mean by $\dfrac{p}{k}$, we already know that its result, $r$, must be such that $r \times k$ will be equal to $p$. i.e. in $\dfrac{p}{k} = r$, $r$ must be such that $r \times k = p$, (e.g. $\dfrac{187}{11}$ is 17 because $17 \times 11 = 187$).

Following this for $\dfrac{3}{\frac{1}{4}}$, we simply seek a result which gives 3 when multiplied by a $\dfrac{1}{4}$.

The question then becomes: *"what times a quarter is 3?"*, or using a familiar rephrasing *"a quarter of what is 3?"* (The answer is of course 12.) In summary

to determine the value of $r$ in $\dfrac{3}{\frac{1}{4}} = r$, find which value of $r$ satisfies $r \times \dfrac{1}{4} = 3$

## Follow-up Exercises

1.    Calculate the value of:

(a)    $4 \div \dfrac{1}{2}$          (b)    $3 \div \dfrac{1}{3}$          (c)    $6 \div \dfrac{1}{4}$          (d)    $10 \div \dfrac{1}{5}$

(e)    $4 \div \dfrac{1}{3}$          (f)    $5 \div \dfrac{1}{4}$          (g)    $20 \div \dfrac{1}{5}$          (h)    $6 \div \dfrac{1}{6}$

2.    Calculate the value of:

(a)    $4 \div \dfrac{2}{5}$          (b)    $3 \div \dfrac{3}{4}$          (c)    $10 \div \dfrac{2}{3}$          (d)    $4 \div \dfrac{3}{4}$

(e)    $6 \div \dfrac{3}{5}$          (f)    $1 \div \dfrac{2}{5}$          (g)    $7 \div \dfrac{5}{8}$          (h)    $20 \div \dfrac{4}{5}$

## Answers

1.    (a) 8     (b) 9     (c) 24     (d) 50     (e) 12     (f) 20     (g) 100     (h) 36

2.    (a) 10     (b) 4     (c) 15     (d) $\dfrac{16}{3}$     (e) 10     (f) $\dfrac{5}{2}$     (g) $\dfrac{56}{5}$     (h) 25