4: Decimals and their Equivalent Fractions

***Question*: What is the equivalent fraction for the decimal 4.422**

Misconception	Correct
The common misconception here is that decimals and fractions are different types of numbers. Hence there is no equivalent fraction for this or any other decimal.	You can express the decimal as 4 and (the fraction) $\frac{422}{1000}$ or as $\frac{4422}{1000}$. These are fraction equivalents. These can be simplified by dividing both numerator and denominator by 2 to give $4.422 = \frac{2211}{500}$ or $4\frac{211}{500}$

Further Explanation

The method above shows how to obtain an equivalent fraction from a decimal. It is even easier to find the decimal equivalent of a fraction. All you have to do is to use division.

So, for example, the decimal equivalent of the fraction $\frac{1}{4}$ is 0.25 as 1.00 divided by 4 will give this result. Similarly the fraction $\frac{3}{5}$ can be expressed as a decimal by dividing 3 by 5 to obtain 0.6

A similar way of achieving this is to find an equivalent fraction that has either 10 or 100 or 1000 etc. as the denominator. In this way we can determine the fraction equivalent from the resulting numerator. For example,

$$\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10}$$

and so the decimal equivalent is 0.6.

Similarly for more complicated fractions such as $\frac{3}{40}$. We can find its decimal equivalent by multiplying both numerator and denominator by 25. This gives

$$\frac{3}{40} = \frac{3 \times 25}{40 \times 25} = \frac{75}{1000}$$

and so the decimal equivalent is 0.075

If you are still feeling confused about equivalent decimals and fractions, then think of a number line in which all numbers are represented. A small part of such a number line is shown below with numbers shown as **decimals** above the line and **fractions** below the line

0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	$\frac{1}{10}$	$\frac{2}{10}\left(=\frac{1}{5}\right)$	$\frac{3}{10}$	$\frac{4}{10}\left(=\frac{2}{5}\right)$	$\frac{5}{10}\left(=\frac{1}{2}\right)$	$\frac{6}{10}\left(=\frac{3}{5}\right)$	$\frac{7}{10}$	$\frac{8}{10}\left(=\frac{4}{5}\right)$	$\frac{9}{10}$	1

You can readily see, for example,that $0.2 = \frac{1}{5}$ and $0.5 = \frac{1}{2}$.

Note: What happens when the fraction cannot be expressed as an equivalent fraction with a numerator of 10 or 100 or 1000 etc.?

An example of this is the fraction $\frac{1}{3}$. You can though divide 1.0000000......... by 3 to give 0.33333.......... This is called a *recurring* decimal, and is often denoted by putting a dot on top of the number (or numbers) that repeat. So we can write

$$\frac{1}{3} = 0.\dot{3}$$

There are many other fractions that give rise to recurring decimals, often with a group of digits that repeat. For example

$$\frac{1}{7} = 0.14285714285714....... = 0.\dot{1}\dot{4}\dot{2}\dot{8}\dot{5}\dot{7}$$

Follow-up Exercises

1. What are the equivalent fractions for the decimals given below?

(a) 0.4 (b) 0.6 (c) 0.12 (d) 0.25 (e) 0.125

(f) 1.75 (g) 3.24 (h) 21.25

(i) 5.875 (j) 4.55

2. What are the equivalent decimals for the fractions given below?

(a) $\frac{3}{4}$ (b) $\frac{3}{20}$ (c) $\frac{4}{5}$ (d) $\frac{7}{25}$ (e) $\frac{9}{40}$

(f) $\frac{23}{20}$ (g) $\frac{57}{200}$ (h) $\frac{23}{500}$ (i) $\frac{223}{25}$ (j) $\frac{3}{400}$

3. What are recurring decimals for these fractions:

(a) $\frac{2}{3}$ (b) $\frac{1}{6}$ (c) $\frac{3}{11}$ (d) $\frac{5}{9}$ (e) $\frac{4}{7}$

Answers

1. (a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{3}{25}$ (d) $\frac{1}{4}$ (e) $\frac{1}{8}$ (f) $\frac{7}{4}$ (g) $\frac{81}{25}$

   (h) $\frac{85}{4}$ (i) $\frac{47}{8}$ (j) $\frac{91}{20}$

2. (a) 0.75 (b) 0.15 (c) 0.8 (d) 0.28 (e) 0.225

   (f) 1.15 (g) 0.285 (h) 0.046 (i) 8.92 (f) 0.0075

3. (a) 0.666....... (b) 0.1666..... (c) 0.272727...... (d) 0.5555........

   (e) 0.57142857142857.............