

Welcome to ACARA Preparation Pack 2:

These materials have been designed to help school administration teams determine how prepared teachers are to implement the major changes that the Australian Curriculum: Mathematics will bring.

This Pack Contains:

How our jobs as teachers have changed: Our role as teachers has had a few significant shifts with the introduction of the proficiency strands. This section explains in a single page what our teaching should look and feel like.

What a week looks like: How to create a weekly timetable to make sure that both the content and proficiency strands from ACARA are covered in the right proportions

What a lesson looks like: Five sample lessons all crossing multiple grades, which are aligned to both the content and proficiency strands for ACARA and are also able to serve as assessment items

Assessment and moderation ideas: Manageable ideas for assessing students on Problem-Solving, Reasoning and Understanding as part of normal teaching

Simple classroom strategies to support problem-based teaching: Five simple ideas to set up in your classroom to help support your class during the transition to problem-based teaching

Managing Differentiation: How to cater for students working at very different levels without driving yourself nuts

Improving Student Reasoning: Simple descriptions of what to look for when assessing Reasoning and Understanding as well as sample questions to ask and tips for helping students to improve

How to use this pack:

- Read through the section on [How our jobs as teachers have changed](#)
- Use [What a week looks like](#) to consider your current practices
- Trial problem-based teaching using one or more of the [five lessons supplied](#). Determine how effective these lessons were and compare them to your normal practices.
- Make sure that you understand the assessment changes by reading through and trialling some [Assessment and moderation ideas](#)
- **Make the decision** about when you would like to implement ACARA and problem-based teaching in your school. **Consider your existing resources** using the tool in ACARA pack 1 and determine if you will need additional help. Now is the time to get help and get ready for next year. Get started by reading [So... where to next](#).
- Once you begin implementing problem-based teaching, implement the [Simple classroom strategies](#) one at a time and make sure that you [Manage differentiation](#).
- Once you have made a good start, target [Improving student reasoning](#).

Attention Classroom Teachers

Back-to-Front Maths is a supportive program written specifically to meet the requirements of ACARA. To find out more, please email ACARApack@kennedypress.com.au or visit us online at www.backtofrontmaths.com.au/teachers or contact your local branch of AAMT.

A Word From the Author

As a mathematics consultant working with schools throughout Australia to improve problem-based teaching methods, I have found that many schools are struggling with similar issues around implementing ACARA. While it is true that there are some content alignment changes, the biggest change for most schools has been the introduction of the Proficiency Strands. In the absence of other guidance, many schools currently focus most of their time, effort and resources into developing *Fluency* at the expense of *Problem-Solving, Reasoning* and *Understanding*.

I have designed this guide to help schools unpack and come to grips with the changes in ACARA. The principles, assessments and observations in this document are the result of long experience consulting in **real classrooms** and have been **field tested successfully in over 500 classrooms across a large cross-section of schools**. I hope that this pack will help you to appraise your current position and begin the process of making change at a structural level.

If you would like information about our workshops, training programs or resources, please send me an email: tierney@kennedypress.com.au.

Kind regards,

Tierney Kennedy



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How our jobs as teachers have changed:

The Australian Curriculum places a very heavy focus on deep-level understanding and mathematical reasoning. This means that students can no longer simply answer routine mathematical questions that look exactly the same as those they have seen before. These type of questions do not require deep-level understanding, logical reasoning, problem-solving or analytical thinking, but instead use only recall of memorised formulae or algorithms without any challenging thinking on the part of the student.

It means that most maths classrooms need to change, and change significantly.

In many traditional mathematics classrooms teachers present an algorithm or formula to students, demonstrate how it works and explain its use, and then give students routine questions to practice until they have the new method memorised. In this environment there is almost never “*inquiry and active participation in challenging and engaging experiences*”, and it is therefore impossible to meet the new requirements.

The new requirements are best met through student inquiry and active participation in a problem-based teaching environment. Problem-based mathematics engages students through challenging problems, and leads them through the process of experimenting to develop new mathematical understanding by focusing on fundamental mathematical principles and patterns.

So, what does this look like?

In problem-based teaching it is incredibly important that teachers don't simply “tell students how to get the answers”, or they lose the opportunity for students to develop their own capacity for logical reasoning and analytical thought.

Instead, teachers have a very different role.

Firstly, they start by asking students a problem that they don't yet know how to solve! This requires the students to think mathematically, and experiment to try and work together to figure out a solution.

Secondly, they look out for student misconceptions (where the student has a fundamental misunderstanding of a concept), and help the students to analyse these ideas to see if they really work. When students self-correct their misconceptions, their mathematical understanding deepens and they learn concepts far more quickly.

Thirdly, and possibly most importantly, the teacher's primary job becomes to ask really great questions that encourage students to think deeply about a problem, access their prior knowledge about it, experiment with different ideas and then analyse how well these ideas work.

Teachers help students to focus on the fundamental principles and patterns in mathematics, therefore enabling deep-level understanding to develop without as much need for repetition and memorisation.

Finally, explanation of algorithms and formulae follow the inquiry process, and are cemented through routine, application and non-standard problems before progressing to a new topic.

What a week looks like:

How to create a weekly timetable to make sure that both the content and proficiency strands from ACARA are covered in the right proportions

There are many different ways of managing the changes, but our preferred model, which is simple to follow and implement, is described below. This plan is based on having at least three separate hour-long blocks and two other shorter blocks (30-45 minutes) for mathematics each week. We have found that this set-up is manageable for most teachers in an average week once time is lost for music, PE and LOTE. There will be three parts to the average week using this breakdown:

1. One of the days should be used at a problem-solving lesson, introducing new content through a novel problem. This problem should be adaptable to suit students working at different levels so that all students have the chance to solve a problem at an appropriate content level. The lesson should focus heavily on the Problem-Solving strand with Reasoning and Understanding strands also playing an important role. Journal problems from *Back-to-Front Maths* are an ideal source of problems for these lessons, and are designed for students working at many different levels including multi-age classes.
2. Two of the days should be used to focus heavily on building the students' understanding of mathematical content through questions that focus on underlying mathematical principles and patterns. These lessons should lead students to work out something new by connecting what they already understand to the new situation, including extending their knowledge to non-standard situations. These lessons focus on the Understanding strand, with some Reasoning and Fluency. Blast activities from *Back-to-Front Maths* for years three to seven are ideal for this purpose as are the Application questions, hand-on activities and games and the manipulation and backwards problems from years one and two.
3. Two of the days (the shorter lessons) should be spent practicing existing skills, rehearsing strategies and revising previously-covered content. These lessons build Fluency. Examples of these activities are included in the Teaching Resource Books and Planning Proformas for *Back-to-Front Maths* for years one and two. Investigations may also be useful for this purpose and are included for years 3-7.

Please note that your break-down does not need to involve whole-lessons, this is just a simply way of explaining the allocated time. If your students work better in half-lessons or rotation groups then use that format instead.

Back-to-Front Maths provides examples of using each of these formats effectively with all five lessons.

We have found that having a focus content strand for the three teaching lessons allows teachers to build a more solid understanding of content than having entirely unrelated lessons. This helps to make learning more efficient over the course of a year. The two days for practicing existing skills can be used for a different content strand if you would prefer.

What a lesson looks like:

Five sample lessons all crossing multiple grades, which are aligned to both the content and proficiency strands for ACARA and are also able to serve as assessment items

The following lessons, taken from *Back-to-Front Maths* provide great examples of how ACARA can look in a classroom. They each focus heavily on the Proficiency Strands of Problem-Solving, Reasoning and Understanding as well as teaching the content. Each lesson can also be adapted to multiple levels or grades and is also able to serve simultaneously as learning and assessment for Problem-Solving, Reasoning and Understanding.

Each of the lessons below is taken from year 3 or 4 of *Back-to-Front Maths*. Each consists of a journal problem and lesson plan. The lesson plans explain how problem-based teaching would look in this lesson as well as containing suggestions for support students, and opportunities for extension. Each lesson could therefore be adapted from years 1-7 or used with multi-level classes.

Lesson 1: Number lines

Lesson 2: Place value and regrouping

Lesson 3: Measuring area

Lesson 4: Time

Lesson 5: Likelihood

Lesson 1: Ordering and counting numbers to 1000

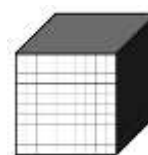


Year 3


Stretch a 10 metre long piece of string across your classroom. Put 1 block MAB on one end and a 1000 block MAB on the other end.


Where would 100 go? Where would 200 go? Where would 10 go?

Draw your completed number line here once you are happy with it.



Make the following numbers out of MAB and work out where they would go on the string: 100, 10, 50, 500, 300, 650, 8, 704 and 321. Add the numbers onto your line drawn above.

 **Sharing time:** Explain how you worked out where 100 would go:

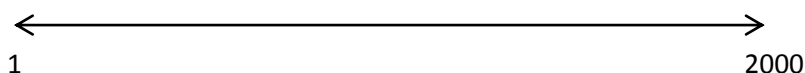
 **Understanding:** What number is half way to 1000? So what does this mean in terms of your number line?

Manipulation problems:

Level 1: Where would the number 905 go? Add it to your number line.

Level 2: Below is a number line from 1 to **2000**.

Put an **X** where 500 would go and a **★** where 1000 would go.



Teacher initials:

Date:

Problem solving / T&R:

- Problem solved with minimal or non-mathematical prompting
- Some leading questions were used to prompt thinking
- Solved after explanation
- Did not work out solution
- N/A- not a novel problem

Reasoning / Comm.: (verbal, written, working and equations, or visual representations)

- Clearly and logically reasoned
- Easily understood
- Understood with some interpretation needed
- Some gaps but on topic
- Minimal or off topic

Understanding / Reflect:

- Connected manipulation problems to previous questions and answered easily
- Connected manipulation problems to previous questions with some prompting, and answered correctly
- Answered once the similarities to previous questions had been pointed out
- Had some problems in answers but was on the right track
- Did not answer appropriately
- Student not observed

Journal problem 3, year 3: *Ordering and counting numbers to 1000*

Introduction:

Be aware that students may take a significant amount of time just to work out where each hundred is on their number line. Try to encourage students to guess or estimate the position first and then see how close they are. You may wish to cut out the larger numbers, and just start out by making a number line from 0 to 200 (see Blast activity A1). Be aware that you do need to progress to larger 3 digit numbers at some point. You will probably want a digital camera in order to photograph student solutions. Complete Blast activities A1-A5 before beginning this journal problem.

You will need: 10m lengths of string, MAB and measuring instruments for this activity.

Leading questions:

- Where do you think the zero would go? How about the 1000? If I was to put 500 on the number line, how far along the line do you think it would be?
- How long is the string altogether? What size number does it represent? So if I was to find 500, where do you think it might go? How could you work it out?
- How long do we need for each hundred? How long do we need for each... ten or one? So how long do we need for...?
- Teacher marks on 100. How could you work out where 200 would be? 300? 400?
- Teacher marks on 10. How could you work out where 20 would be? How about 110? How about 120?

Misconceptions to watch out for:

- Students who try to put 100 in the middle instead of 500. Lots of students tend to think that 100 is half way between 0 and 1000 rather than one tenth of the way.
- Students who do not realise that each 10 is the same distance away from the previous hundred (e.g. 80 is the same distance from 0 as 180 is from 100).

Teaching Tips:

- **Support students or lower grades:** follow the last two prompts above to make a number line that adds on hundreds (e.g. make 1-100, then 101-200 and join it on. Keep going til you get to 1000). For lower level students try a number line from 1 to 10 to see if they have the concept of even spacing for ones.
- Lining up MAB against the string will show you where each number should be. Try lining up 100 MAB (10 tens blocks) to show students where 100 is on the line (at the 1m point). They can repeat this process to get to 200 if needed then will hopefully see the patterns. Alternatively, line your metre ruler up along the string. You can use 10 metre rulers to represent the whole line.

Follow up ideas:

Choose any other whole number up to 3 digits to place on the line. Reinforce using 100s chart and number line activities in Blasts books. Work in groups as appropriate. Follow up with Blast activities A6 and A8.

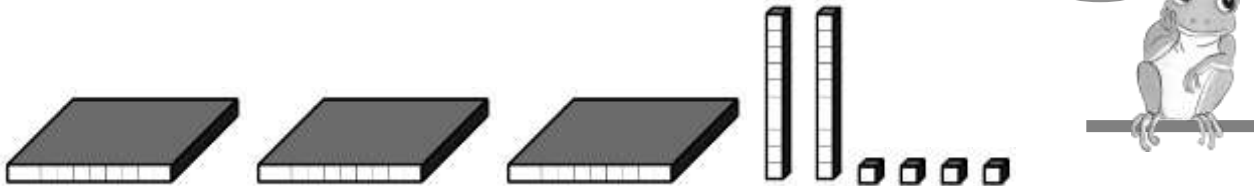
Suggestions for making the problem more difficult:

1. Change the line from 0 to 1 million
2. Start at 200 and go to 1800
3. Start at -100 and go to 100
4. Start at 0 and go to 1 (use decimals or fractions)

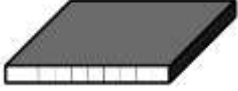


Lesson 2: Regrouping numbers to 999

What different ways can the number 324 be made if you were using hundreds, tens and ones MAB? Make it as many different ways as you can and fill in the table below to show which blocks you used.

One way that you could make 324 is:



Write five other ways that you could make 324 in the table:

	How many hundreds do you have? 	How many tens do you have? 	How many ones do you have? 
1			
2			
3			
4			
5			

Sharing time: What patterns did you find?

Understanding: Which of the numbers in your table is the biggest?

Explain your answer:

Expanded notation:

If I was writing the number 324 using expanded notation I could write:

$$3 \times 100 + 2 \times 10 + 4 \times 1$$

How else could I write the number 324 using expanded notation? Refer to your table on the previous page if you are stuck for ideas.



Sharing time: How did you come up with your solutions?

Look at your solutions. What patterns can you see?



Understanding: How do you know that your answers are right?

Manipulation problems:

Level 1: Make 602 in at least 2 ways. Write the number in expanded notation:

Level 2: Write as many ways to make 602 as you can in expanded notation: Use a separate piece of paper if you need to.

Teacher initials:

Date:

Problem solving / T&R:

- Problem solved with minimal or non-mathematical prompting
- Some leading questions were used to prompt thinking
- Solved after explanation
- Did not work out solution
- N/A- not a novel problem

Reasoning / Comm.: (verbal, written, working and equations, or visual representations)

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Understanding / Reflect:

- Connected manipulation problems to previous questions and answered easily
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Journal problem 4, year 3: *Regrouping numbers to 999*

Introduction:

This activity encourages students to make the number 324 in as many different ways as they can. Students swap the MAB as long as the total number remains the same. The aim is for them to see that one number can be made using different combinations of ones, tens and hundreds. Complete Blast activities A1-A5 before beginning this journal problem.

You will need: You will need substantial supplies of MAB and calculators for this activity. One way to accommodate this is by completing the activity in 'rotational groups' time.

Leading questions:

- Is there a different combination of blocks that you could use so that you would still have 324?
- How many hundreds do you have? If you only had 2 hundreds blocks, is there a way that you could use other blocks as well so that you could still make 324?
- Let's just look at the 24. How many tens blocks are there? How many ones blocks are there? Is there another combination of blocks that you could use to make 24?
- Teacher makes 1 ten and 14 ones on one side and 2 tens and 4 ones on another side. Are these amounts the same? How do you know?

Misconceptions to watch out for:

- Students who think that you can just swap the digits to any column (e.g. instead of making 324 they make 234 from the blocks). This shows a fundamental misunderstanding of place value and you will need to make sure that this is not a problem for other students.

Teaching Tips:

- **Support students and lower grades:** Don't try non-standard grouping of 3 digit numbers until the students can regroup for 2 digit numbers using non-standard groupings. For a lower level again, do partitioning of single digit numbers (e.g. how many ways can we make 6 using unifix cubes?)
- Make a number in one way first (standard way), then ask students for one other way of making the number. Build up one by one until you have noticeable patterns. You may wish to use a table to summarise these (see student Blasts book).
- Use pictures of the blocks to put on the board so that students can remember the different ways and associate names with pictures and symbols.
- The manipulation question has an internal zero. Some students may need help to work out that they can swap a hundreds block for tens then trade these tens for ones. Try to encourage students to work this out for themselves.

Follow up ideas:

Choose any other 3 digit number (without internal zeroes) to make in multiple ways. Challenge students to work out an answer first and then check it using the blocks. Follow up with Blast activities A7 and A9.

Suggestions for making the problem more difficult:

1. Use more digits and don't make it using blocks
2. Give students a pile of blocks that they need to start with. They can add other blocks to that pile.
3. The student must make the number, but using three identical piles of blocks
4. Involve a decimal, e.g. make 23.4 as many different ways as possible
5. Involve a negative number or a fraction (e.g. make $\frac{3}{5}$ as many different ways as possible)

Lesson 3: Area



One of the other classes in your school has complained that your class has too much space. They think that it is not fair that their class has less space than your class. Work out whether they are right or not.

What do you think the problem is asking you to do? Come up with a plan for how to solve the problem. Explain how you will measure the space.



How big is your classroom? Show how you worked it out here and write an explanation. (Teacher: attach a photo if possible)

Which class has the most space? How can you be sure?



Sharing time: How did you come up with your solution?



Understanding: Prove that your answers are right.

Complex manipulation problem:

The class argues with your findings to say that even though the space is not that different, they have less space **per student** than you do. Work out if they are right or not.

Teacher initials:

Date:

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Journal problem 21, grade 3: Area**Introduction:**

Complete Blast activities E5 and E8 before beginning this journal problem.

You will need: A variety of instruments on hand that students might choose to use to measure area (e.g. length measuring instruments, large pieces of news paper).

Leading questions:

- Do you think that the other class is concerned about how high our classroom is? Well what should we be measuring then? (floor space)
- How could we decide who has the biggest floor space? How could we measure floor space? What could we use to measure it?
- Could we work it out if we didn't have anything on the floor? What would we do?
- Let's use these pieces of paper to measure the floor space. How could we do it?
- Is there a way that you could just cover part of the floor and then use this to work out how many would be needed for the rest of the floor?
- What are you going to do if the paper doesn't fit neatly into the space (hang over the edges)?
- Is it ok if we have these pieces of paper not touching? What about over here where they are overlapping?

Misconceptions to watch out for:

- Students who decide that the biggest classroom is the longest rather than the one with the most floor space (area).
- Watch for gaps or overlaps if the students are using paper to measure area.

Teaching Tips:

- You may need to suggest that they count the number of pieces that fit across one dimension, and the number in the other direction. Draw this on the board and get them to fill in the missing pieces and count them up.
- Hopefully students will work out that they can skip count to find the area, or multiply the length by width.

Follow up ideas:

Give other rectangles for students to find the area of (e.g. the top of their desk). Use standard or non-standard units.

Suggestions for making the problem easier:

1. Measure length instead of trying to calculate area. This can be done using metres or informal units.
2. Consider informal aspects, such as how many desks fit in the room and if there can be as many children sitting on the carpet.
3. Choose your rooms carefully so that they are rectangular or square, with clear space along the walls or an empty floor

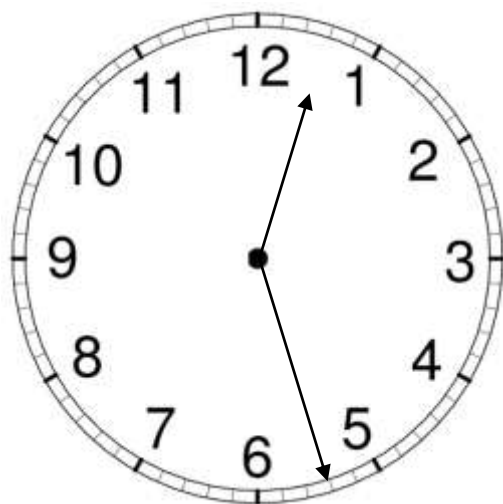
Suggestions for making the problem more difficult:

1. Create a scale model or drawing
2. Use area formulae for regular shapes
3. Create rates of people per metre or metres per person
4. Measure length of each wall in metres, including using decimals
5. Take the furniture in the room into account and just consider the bare floor space

Lesson 4: Elapsed time



Paul's dad needs to leave home **1 hour and 15 minutes before** school ends so that he gets to school in time to pick Paul up. School ends at 3:05pm. Look at the time on the clock below. How long does Paul's dad have before he has to leave?



1. What time is it now?
2. What time does Paul's dad have to leave?

Communicating:

Explain how you worked out what time Paul's dad had to leave:

3. So how long is there between now and when he has to leave?

Communicating:

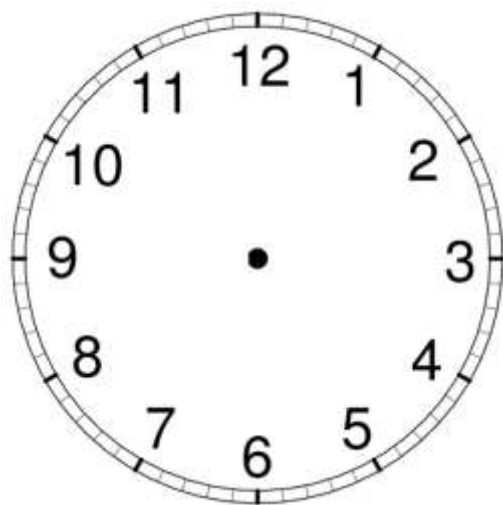
Explain how you worked out how much time there is between now and when he has to leave:

Manipulation problems:

Paul's dad needs to go to the shop on the way to school. It will take him **half an hour** to do the shopping. **What time should he leave home** so that he has time to do the shopping?



Draw the time on the clock.



Explain how you worked out what time Paul's dad had to leave:

How long does Paul's dad have before he has to leave home if he wants to do the shopping before picking Paul up?

Explain how you got your answer:

Paul's dad accidentally fell asleep! The time when he woke up is shown below. Does he still have time to do the shopping if he leaves now?

**Understanding:**

Prove that you are right. Show how you worked it out.

Teacher initials:

Date:

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Journal problem 23, grade 4:*Elapsed time***Introduction:**

This Journal problem is a multi-step problem. If students have trouble with the multistep part, just have them work out what time Pauls' dad needs to leave. Complete Blast activities F1 and F2 before beginning this journal problem.

You will need: A clock with movable hands for students to use if they are stuck. One is included on the next page to photocopy and cut out.

Leading questions:

- Which time do we need to find first? How can we do that? Then what do we do from there?
- What time should he be leaving? So how far away is that from the time shown?
- Let's make the times on this clock. Where do we start? Now how long do we have before he has to leave?
- We've already worked out that he needs to leave at 2:15pm, so how far is that away from 12:30? Is it more than an hour? Is it more than two hours?

Misconceptions to watch out for:

- Students who try to use 100 minutes in an hour instead of 60 minutes.

Teaching Tips:

- **Support students and lower grades:** Change the leaving time to "one hour before" and the times that are at whole or half hours only. Use the clock with movable hands to count backward by one hour, then determine how much time there is between 12:30 and 2:00.
- Extension students: move to the Manipulation Problems fairly quickly. Continue altering the scenario using "what if" questions.
- You may need to prompt students to draw the "leaving time" on another clock and then compare the two clocks.
- Check whether students can count in fractions of hours, or whether they get stuck and need to go back to smaller time frames.

Follow up ideas:

Follow up with Blast activities F3 – F5.

Suggestions for making the problem more difficult:

1. Add in extra complexity such as additional stops to make on the way (petrol, shopping)
2. Alter the times to not be 5 minute intervals
3. Cross time zones (e.g. Gold Coast in QLD to Coolangatta in NSW)

Lesson 5: Rolling a Die



Find a partner. You may each choose three numbers from a die (so that the six numbers are split between you). Roll the die 30 times, scoring a point each time one of your numbers comes up. Who do you think will win?

What numbers have you chosen?

What numbers has your partner chosen?

Who do you think will win?

How will you know who has won?

How can you keep track of how many points you each have?



Try out the game here:

Who won?

Was it fair? Explain your answer:

Game two: Now take two dice. Your score is determined by adding the numbers on each of the dice together (e.g. 5 on one die and 2 on the other die would score 7). Choose three scores that you think will be likely to help you to win. Collect one point for each time your score is rolled. Conduct your experiment and record the results here:

 **Understanding and Communicating:**

What did you find? Were you surprised? Do you think that this would always happen, or is it a matter of coincidence? Explain your answers:

Manipulation problems:

Level 1: If three people were playing the original game instead of two people, and each person got to choose two numbers instead of three, how many points do you think each person would have?

Level 2: Write down all the possible scores from game two. Which score would be the best ones to choose?

Teacher initials:

Date:

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Journal problem 24, year 4: *Rolling a die***Introduction:**

This aim of the first activity is for students to realise that rolling any number on a die is equally likely, so it will not matter which of the three numbers each player chooses. In the second activity it is the sum of the numbers rolled by two dice that creates a score, so students need to consider what the numbers on the dice are, and hopefully work out that the most likely score to roll is 7 (highest on one die + lowest on the other die = most common score). This activity may not be suitable for support students. Complete Blast activities I1 – I3 before beginning this journal problem.

You will need: Dice for this activity (two between two or three students).

Leading questions:

- How would you know who the winner was?
- How could you record the scores?
- Have a try. Now let's work out how to keep track of this game.
- How would you know which scores you should pick for the Understanding problem? What are all of the possible scores? Which one do you think would be most likely to occur? Why?

Misconceptions to watch out for:

- Students who do not think that there is an equal chance of the die landing on any one number.

Teaching Tips:

- **Support students and lower levels:** teacher becomes one of the players in the pair/three so that they can 'wrongly' record what happens and see if students correct you. For lower levels: Use a spinner that is equally or unequally coloured into two colours.
- Consider as a follow-up task having students design their own die that is 'rigged'.

Follow up ideas:

- Die with more sides, different colours, same numbers on 2 of the sides.
- Design spinners with equally likely, or not equally likely outcomes. Follow up with Blast activity I4.

Suggestions for making the problem more difficult:

1. Use two die and consider the sum of rolling two die rather than the individual scores
2. Compare having only two rolls each with having 20 rolls each. Consider the impact of number of repetitions.
3. Give students the results of a trial and have them predict what the die looked like
4. Use a die with more sides

Assessment and Moderation ideas:

Manageable ideas for assessing students on Problem-Solving, Reasoning and Understanding as part of normal teaching

Assessment and grading is often one of the most difficult tasks for teachers. The descriptions of the standards can be confusing, and it is important to clarify these as we move to a National Curriculum. In this section you will find information about the standards in Back-to-Front Maths as well as when and how to assess. Further examples of assessment items, such as moderating tasks, can be found at www.backtofrontmaths.com.au

The criteria: what the proficiencies mean

Back-to-Front Maths assesses students using the Proficiencies described in the Australian Curriculum: Mathematics. A brief explanation is included below to help you understand what each of these mean, and therefore what we are looking for when we assess them.

Problem-solving: At an A and B level this requires solving problems that are novel to the student, so that he or she is required to develop their own new strategies from their existing knowledge and skills. The student needs to bring his or her own thinking to bear on the problem, and demonstrate insightfulness. The teacher provides leading questions to help the student to get ideas to try, evaluate their ideas, and bring the student's existing knowledge and skills to the problem.

At a C, D and E level this requires solving application problems that only require knowledge and skills that the student has already rehearsed in class.

Reasoning: This proficiency involves a student proving the correctness of his or her answer. This proof can take many forms, including explanations, logical working, diagrams and drawings, modelling, debate and formal proofs. It can also include finding faults in a student's own or others' working.

Understanding: This proficiency focuses on developing a deep understanding of the patterns and principles that underpin mathematics. A student needs not only to be able to state these patterns, but to be able to explain, find, apply and make connections between these. Deep understanding is often shown when students adapt or manipulate formulae to account for changing or non-standard circumstances. They are able to use what they have already done and build on it, rather than working each new question out from scratch.

Fluency: This proficiency focuses on routine mathematical content and skills. Students need to be able to work accurately and efficiently with complex ideas. The assessment statements within the ACARA documents reflect the content level to which students should be working in each grade.

When and how to assess:

Throughout the year you should assess on numerous occasions. Please find below a suggested schedule for your assessment tasks. You will need to add in your own assessment for Fluency, and also for mental mathematics. At the bottom of the page you will also find a “tick and flick” sheet for assessing any problem-solving task. Following this section you will find detailed explanations for the assessment in Back-to-Front Maths.

Semester 1:

- Early in semester 1 complete the first moderation task. This will give you baseline measurements for students’ proficiencies in problem-solving, reasoning and understanding. It will also help explain the standards to you in a more meaningful manner. This should be formative only, not summative.
- During semester 1 try to examine 5 students per lesson during Journal problems in order to gauge their improvements. These should be formative only, not summative. Examples of these problems are given in the lesson plan section.
- Towards the end of semester 1, mark the last 3-4 Journal problems for each student using the tick-and-flick box. Use these marks in combination with the Blasts book to mark the criteria sheet from ACARA pack 1. You will also need to include some application questions for students who are in the C/D/E category, which may be selected from those suggested in the lesson plans in the Blasts books.
- Towards the end of semester 1 complete the second moderation task.
- Create your own content assessment, such as a checklist, test or observation. This should give you a mark for Fluency.
- **Final grade for reporting:** Compare the results from your criteria sheet and the second moderation tasks to check that they align. If there is a discrepancy, then you will need to use your judgement to grade the student appropriately. Be aware that the moderation tasks only exist to help illustrate the criteria. You may find that you have been marking too easily or too hard, so adjust your marking accordingly.

Semester 2:

- Consider using an investigation throughout the semester and using this as an additional assessment piece. If using these, never use the first investigation as a summative piece as both students and teachers need time to get used to the requirements.
- Continue marking 5 students per lesson on Journal problems as formative tasks.
- Towards the end of semester 1, mark the last 3-4 Journal problems for each student using the tick-and-flick box. Use these marks in combination with the Blasts book to mark the criteria sheet. You will also need to include some application questions for students who are in the C/D/E category, which may be selected from those suggested in the lesson plans.
- Towards the end of semester 2 complete the third moderation task.
- Create your own content assessment, such as a checklist, test or observation. This should give you a mark for Fluency.
- **Final grade for reporting:** Compare the results from your investigations, criteria sheets and the third moderation tasks to check that they align. If there is a discrepancy, then you will need to use your judgement to grade the student appropriately.

Teacher initials:

Date:

Problem solving / T&R:

- Problem solved with minimal or non-mathematical prompting
- Some leading questions were used to prompt thinking
- Solved after explanation
- Did not work out solution
- N/A- not a novel problem

Reasoning / Comm.: (verbal, written, working and equations, or visual representations)

- Clearly and logically reasoned
- Easily understood
- Understood with some interpretation needed
- Some gaps but on topic
- Minimal or off topic

Understanding / Reflect:

- Connected manipulation problems to previous questions and answered easily
- Connected manipulation problems to previous questions with some prompting, and answered correctly
- Answered once the similarities to previous questions had been pointed out
- Had some problems in answers but was on the right track
- Did not answer appropriately
- Student not observed

Assessment tasks in Back-to-Front Maths:

Assessment for Back-to-Front Maths consists of three parts: Journal problems and Blast activities, Moderating tasks and Investigations. Between these students are assessed on Problem Solving (Thinking and Reasoning), Reasoning (Communicating) and Understanding (Reflecting) proficiencies.

You will need to write your own assessment for Fluency (Knowledge and Understanding), because the order in which you choose to cover the content will determine the assessment form and criteria.

A traditional test is an appropriate instrument for assessing Fluency, and should be constructed using a selection of routine questions and application problems (to assess Problem Solving for a C, D or E level). A test does not need to try and assess the other criteria as they are adequately covered throughout this program.

Journal Problems

Most students will not receive a mark on their Journals in most lessons. Aim to mark around 5 students per lesson. You may choose to only look at some of the proficiencies rather than all of them (e.g. just mark Problem Solving for student A, and just mark Reasoning for student B), or you may choose to not mark any students at all during a lesson. Keep the following advice in mind:

1. Only mark a student's Problem Solving if you actually observe them
2. Remember to adjust the problems up and down to suit your students. This does not affect their grade for Problem Solving.
3. Students should work together – just give them another question with different numbers to check if only one of them did the thinking.
4. Only mark a student's Reasoning if they successfully solve the problem.
5. Remember that only the last 3-4 Journal problems in each semester count as summative, so focus on teaching well rather than on trying to assess every student.
6. Remember to use Sharing Time, particularly in lower primary, to mark students' Reasoning and Understanding.

Problem solving:

Use the 'tick and flick' chart on the student Journals to record your observations for students. Choose the statement that best describes the problem-solving attempts by the student.

Problem solving / Thinking and Reasoning:
 Teacher initials:
 Date:
 Student solved the problem with:

- Minimal help
- Some prompting
- Solved after explanation
- Did not work out a solution by themselves
- N/A – not a novel problem
- Not observed today

Indicates that students are demonstrating problem solving at an A or B standard. Minimal help means that teacher help was non-mathematical (e.g. getting started by suggesting to draw a picture). Some prompting means using mathematically leading questions.

Indicates whether students are demonstrating problem solving at a C or below standard (further assessment is needed using application questions).

Indicates whether the problem was not novel for the student (e.g. they already knew the formula to use).

Adjusting the problems:

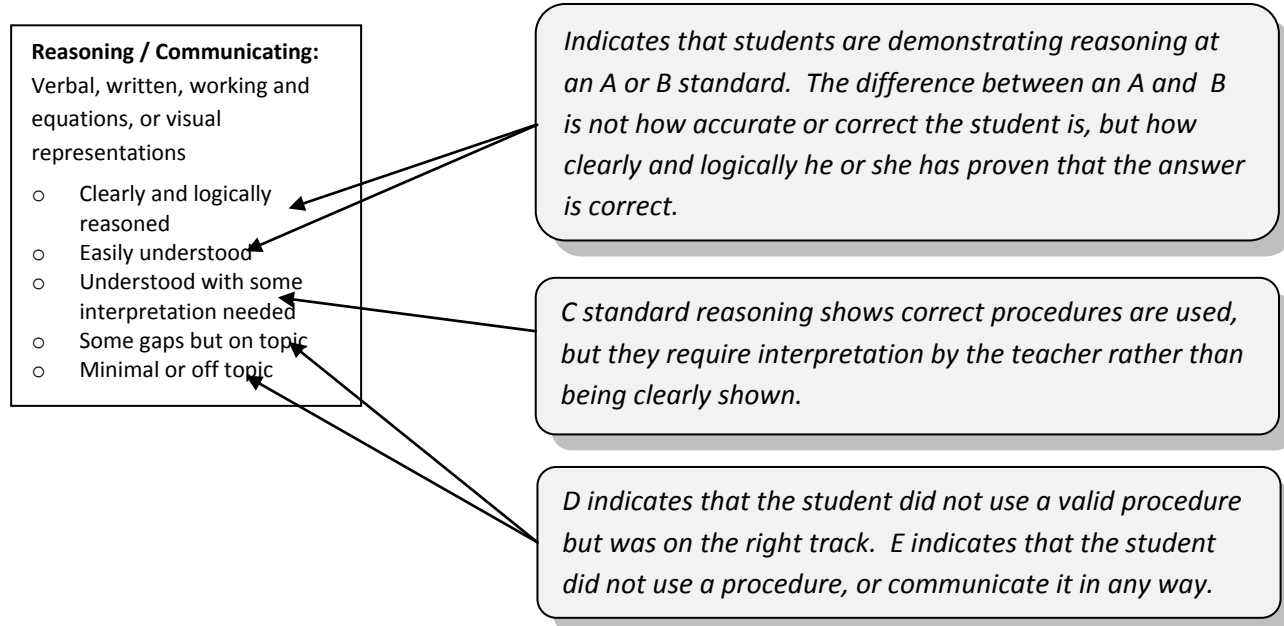
Make sure that the problem being solved is **truly novel** to your students. It needs to be something that requires them to do their own thinking, and develop new or innovative strategies.

If the question asked is too easy for your students, or they already know how to solve it, you have two options. The first is to mark only the “manipulation problems”, or if these are not novel either, choose the box marked “N/A – not a novel problem”. This alteration does not mean that the students should receive a higher grade for Problem Solving. Higher content levels do not equate to higher problem solving. A student who has adjustments made to the problem should still be marked using the same requirements for insightfulness.

If the problem is too difficult for your students because they lack content knowledge you will need to adjust the numbers in the problem (see the lesson plans for suggestions). This adjustment does not lower a student’s grade for Problem Solving, as it only alters the content in the question, not the Problem Solving required. A student who has adjustments made to the problem should still be marked using the same requirements for insightfulness.

Reasoning:

Use the ‘tick and flick’ chart on the student Journals to record your observations for students. Choose the statement that best describes the problem-solving attempts by the student. Make sure that you look at the combination of their working (e.g. writing algorithms), verbal explanations, drawings and models. Their reasoning is formed through all of these.



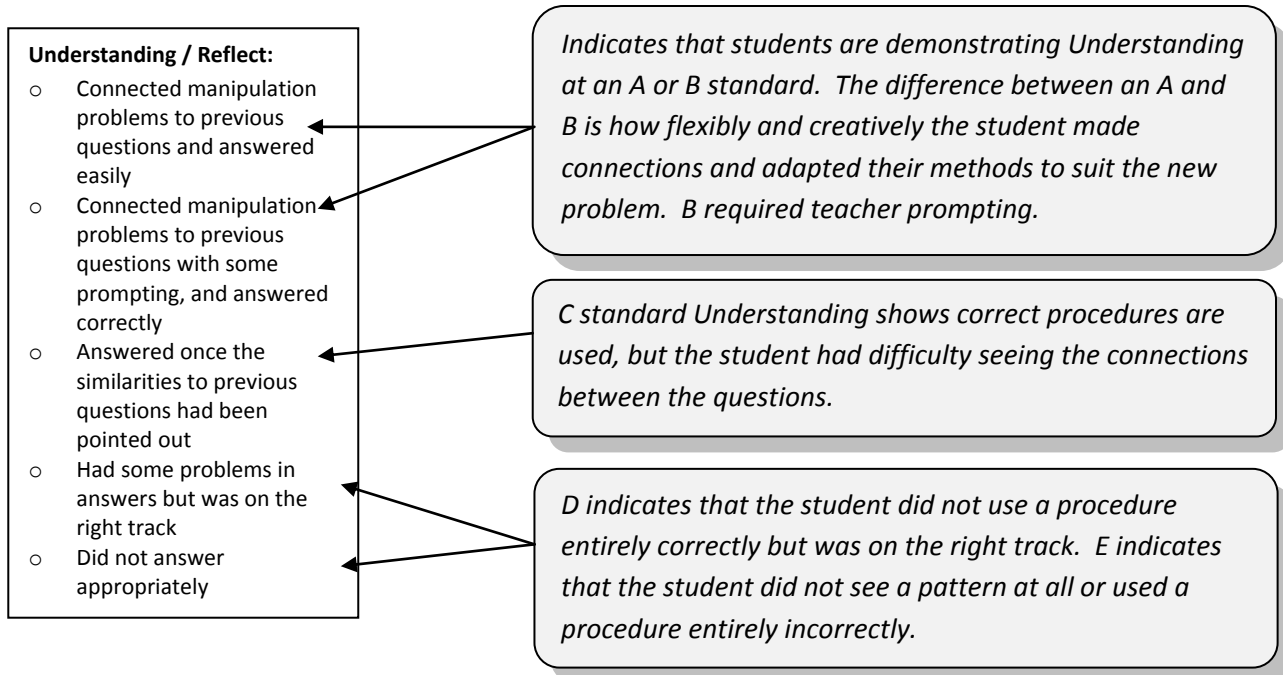
Reasoning can only be marked in a Journal problem if the student successfully solves the problem. If the student does not solve the problem, you will need to mark reasoning using the application problems on their test. The same standards apply as indicated above.

Reasoning does not need to be marked on a student’s first attempt. Reasoning should include a process of presenting, clarifying and adjusting. A student who shows good reasoning will often start an explanation and then realise part way through that it does not make sense, and will need to go away and clarify their thinking before trying again.

Sharing time is a great time to mark Reasoning. Use the [Sharing time sheet](#) to ask questions of specific students. Mark a few students during each sharing time session, and attach the sheets to the students’ journals.

Understanding:

Use the 'tick and flick' chart on the Journal Problems to record your observations for students. Choose the statement that best describes the patterns and connections made by the student. Make sure that you look at the original problem as well as the manipulation problems when marking.



Simple classroom strategies to support problem-based teaching:

Here are five simple ideas to set up in your classroom to help support your class during the transition to problem-based teaching.

Tip 1: Getting prepared before you start a problem

When preparing to teach a problem-based lesson it is important to ask yourself several questions before you start. Make sure that you consider these questions during your planning time:

- What are the prior concepts?
- What are some questions that I could ask to check their understandings?
- What misconceptions could they have? (These are often listed in the lesson plans)
- How can I lead their thinking?
- What could be missing in their understanding?
- How will I know when they've "got it"? What will I see?
- What am I going to do once they've "got it"?

Read through the lesson plans for the Journal problem that you are about to use, and look at the misconceptions carefully. Have a think about what questions you could ask students to help them evaluate their wrong ideas and figure out a new mathematical understanding. Also, make sure that you have prepared to alter your question for support and extension students. Suggestions are given in the lesson plans.

Tip 2: A Challenge Table

Catering for a very diverse group of learners can be challenging for the most experienced teacher. One simple tool that I have developed to help me cater for multi-levelled classes is a challenge table.

A Challenge Table is a spare table or set of desks somewhere in your class that is set aside as a work space. I use it to invite small groups of students to work with me for five minutes during our problem-solving time. I find that having this dedicated space allows me to maintain a great deal of flexibility within my classroom.

I invite different students to work with me based on what I need to happen within the whole class. Often I invite students with similar communication styles rather than students who are working at the same levels. Inviting all of the students who are dominant allows them to "fight it out", while freeing up your other groups to think for themselves. Inviting all of the students who are very quiet and just go along with the group under normal circumstances means that one of them eventually has to contribute something to solving the problem!

You can use the Challenge Table in whatever manner best suits your class. Consider some of the following ideas:

- Use it for your support students when you first set a problem so that you can help explain the problem in more depth, or offer a problem with a lower content level
- Use it for your fast-finishers so that they can explain their solutions to each other and decide who is right
- Use it for students who all seem to be experiencing the same difficulty or misconception so that you can work with them in more depth

Who you choose to invite is up to you, but make sure that everyone gets a turn. Working one-on-one with the teacher is a privilege that most students look forward to!

Tip 3: Pausing - making teachable moments in only two minutes

Often as teachers we are very pressed for time. There is a very large temptation to push students quickly through content to make sure that we get everything covered. This however is a false economy, leading inevitably to the requirement for re-teaching the same content. The art of pausing makes a significant difference to our ability to teach for Understanding.

I use pausing when I ask a question to a class and only a couple of kids try to answer my question. There is no point gathering student answers at this stage, as most of the students have not understood the concept or the question.

There are a few stages in pausing, and I would like to explain them in brief here:

1. Firstly, I tell the class that I will let them have 30 seconds to talk to their friend and try to work it out. I look away from the students, so that they realise that now is “talking time” and they are allowed to discuss the question. Usually this just results in the few students with their hands up telling everyone else what they think the answer is. Clearly now is not the time to ask for answers!
2. The second step is to talk through the sub-steps to solving the question very slowly, pausing after asking each sub question. This usually involves breaking the options down to only two (bigger/smaller, further/closer etc.), and using long “hmm” sounds while looking at the board. By taking my time and breaking the question into much more manageable steps I allow students to think the problem through while I lead them by asking simpler questions. During this process I commonly hear “Oh!” shouts across the room as students work out for themselves what to do with the question. When I turn back to look at the class with a confused expression on my face there are usually frantic hands waving by around 80% of the students.
3. Now is the time to ask for student input, but here I do something a little different. If possible, I ask someone in the class who does NOT have their hand up what they think. I deliberately choose someone who I think does not know. This therefore creates an opportunity to talk through the problem and gives the student a chance to learn something new.
4. I ask the student to come to the front of the room and “think it through with me” regardless of whether they give an answer or not, and regardless of whether the answer is correct or incorrect. At this stage I lead them through the whole process again with leading questions, pausing at each stage to allow their thinking to catch up. At some point if they have given me a wrong answer the realisation comes and it shows on their face. This is the time to ask, “So do you want to keep your answer or change your thinking?”
5. Once a student has decided on an answer, we put that number back into the original question and check that it works.

This whole process takes between 2 and 5 minutes, and means that at the end of the time I can always give the response, “Nice thinking!” to a student who otherwise would have been completely lost.

Tip 4: Dealing with students who don't want to try anything: setting the scene for risk-taking

One of the most difficult problems to overcome when establishing a problem-based classroom is encouraging risk-taking behaviour on the part of the students. Unless students are willing to try out an idea and risk being wrong, there will be very little progress. I have found that the further through school students go the less willing they are to take a risk, and the more avoidance behaviours they have developed. The most common avoidance behaviour is simply to do nothing in the face of a problem, and to wait until the teacher breaks down from time-pressure and explains how to use the formula/algorithm.

Risk-taking can be encouraged using a number of strategies. Below I have outlined some of the most common things that I do when implementing problem-based teaching for the first time. These include the initial discussion and scene setting, engaging students and provoking a response, deliberately displaying a [misconception](#), using [metalinguage](#) and combating right/wrong thinking.

The initial discussion and setting the scene:

It is important to discuss your expectations with students up-front so that they know what to expect. I always begin with a class discussion that goes something like the following. I apologise for it being so wordy, but it is difficult to explain without demonstrating.

“You know last year when you did a maths test, were the questions usually completely different from anything that you have ever seen before, or kind of like ones you had done in class? So do you think that your teachers were looking at your “working out a new idea completely for yourselves”, or do you think they were looking at your “remembering what you had been taught”? **Well, today I'm not looking at your remembering. I'm looking at your working stuff out for yourselves.** That means that today our maths lesson is going to be a bit different to what you are used to.

For starters, I can't ask you a question that you already know how to answer. Then you would just be remembering stuff instead of working out something new. **So today I'm deliberately going to try to confuse you!** I don't want to see all the stuff you know – I want to see what you can work out. Who in maths is usually a bit confused? Well you guys have a real advantage today: you know what it feels like to be confused. Today everyone is going to be confused for at least some of the time - even the kids who are usually pretty good at maths. Oh, and if you put your hand up to say, “Mrs Kennedy I don't know what to do”, I'm going to respond with, “That's great! Now you have something to work out!”

Now to show you that I'm serious about looking at your thinking **I'm not even going to mark your answers. I honestly don't care if they are right or wrong, and I probably won't tell you if they are right or wrong. I'm looking at your thinking, and that is far more important.** You know sometimes people can get the right answer, but have totally wrong reasons or no reasons at all. I'm going to mark that as totally wrong. And sometimes you can have really great thinking, but you just added up a bit wrongly so your answer ends up being wrong. I'm going to mark you as totally right. I'm marking the thinking and the reasons, not the answers.

Let me explain. **Today, you are going to be wrong. I will keep on going until you are wrong at least once.** Say I ask you a question, and remember **I am deliberately tricky and weird**, and you have no idea at all and you come up with something that is totally wrong. Then I ask you some questions, and you do a bit more thinking, and you have another idea. This one is wrong too, but it's getting closer and you've done some great thinking. Then I ask you some more questions and you do some more thinking and try out something else. You're still wrong, but you're getting really close now. Then you have this moment where you think, “oh, now I get it!” and you work it out. Do you really need me to tell you that you are right? And can you see how much great thinking you did along the way to get there? That's what I'm looking at.

This brings me to one of my rules: **you can't use a rubber today.** Imagine what would happen if every time you tried something that turned out to be wrong you rubbed it all out! How would I see all that great thinking? Then I would only have an answer, and I don't care about the answer anyway. **Today you are allowed to change your mind as many times as you want. Just don't rub it all out.** Instead, just rule a line and start again underneath. Once you're happy with your final answer put a big star next to it. Also, I need to explain that I don't have x-ray vision – I can't see what you are thinking unless you explain it to me or show it to me on paper. Now I don't care about your spelling, or how beautiful

your writing is, but I do need to see your thinking. So show me your thinking on paper. Write down all your ideas no matter if they are right or wrong.

I have another rule too. Who does their best thinking with a friend? **Well today you are allowed to work together, as long as you are both thinking. In fact I don't even care if you just copy down exactly what they have written down.** It's not really going to help you though – am I marking your answers? No, that's right, I'm looking at your thinking. So if you and your friend have the same idea, then write that down. But if you have a different idea, then try that instead. That way you can try out both ideas and learn together.

You know how sometimes when teachers say that they want you to work it out using any way you want but they really mean that they want you to work out what's in their heads? Well, when I say I want to see your thinking I actually mean it. **I really want you to try out your own ideas. If you don't have any ideas, that's ok, just have a guess. That's trying something and that counts as thinking. I don't care if it's wrong. I care that you tried,** and you know you might just work it out by guessing!

One more thing that I need to let you know. You know how sometimes teachers ask you questions like, “Are you sure about that? Can you just check that one for me please?” and what they really mean is, “That one is totally wrong”? Well I'm going to ask you questions like that even if you're right! I actually want to see what you think, so I am deliberately tricky. I can even use my voice to make the wrong answer sound like it's the right one: “When you add two and three together, is the answer five (sound uncertain) or is it four (sound certain)?” I really do want you to have to think about it. Oh, and if you already know the answer please let me know so that I can make it trickier for you, or else you'll miss out on a chance to show me your thinking.

Ok, so now that's all out of the way, **are you ready to be confused? Great!** Let's get started.”

Engaging students and provoking a response:

I find that playing a character is often important in getting students to “lighten up” and enjoy maths. In many maths situations in primary school playing is low on the agenda, and doing things “the right way” is emphasised. It is essential to turn this around if you want your students to really engage with a problem and experiment with ideas. Often I am deliberately overly theatrical or a little weird in order to provoke a response from students who would otherwise try to ignore me.

Deliberately displaying wrong thinking or a misconception:

Students are much more likely to show you their own misconceptions if you pretend to have one yourself. I will often tell students that what they are doing with that formula doesn't make any sense at all, and then demonstrate a misconception. This challenges students to check and see if their thinking does make sense, and to prove that my misconception is actually wrong. This is especially helpful for engaging both students who have a misconception, and students who really know that I'm wrong. It's always fun proving the teacher wrong!

In the same manner, I always try to get students to have a guess at what they think the answer might be before we get stuck into serious problem-solving. This usually takes the form of a class vote. I always put my hand up for every option and make sure that they all sound equally valid.

Combating right/wrong thinking:

Many students firmly believe that there is really only one way to solve a problem, and that it is the way that the teacher is about to show them. All other methods are wrong, or if not wrong at least not as valid as the teacher's method. They see it as a total waste of time trying to work out something themselves instead of waiting for the teacher to show them how to do it. To combat this thinking, we need to help students think about maths as something other than the duality of right/wrong.

Most students don't think about how mathematical ideas and theories are formed in the first place. Formulas don't just exist - people work them out by experimenting with different ideas and seeing what works. Formulas are a summary of the mathematical thinking that someone did. They are certainly useful, but are in no way what true mathematics is really all about. True mathematics is about using what you already know to work out something that you don't know yet. It is about finding patterns and connections between ideas. It is about experimenting and finding out what works, and then

summarising the principles involved so that they can be used in other situations and by other people. If we could only use existing formulas, then nobody could work out anything new.

Tip 5: Managing group work

I am often asked in professional development sessions what model of group work to use in problem-based lessons: mixed ability or like ability groupings. My answer to this question is somewhat more practical. I have found that in most classrooms we actually have to use behaviour groupings: students are matched up according to who won't kill each other. This is a pretty workable solution in problem-based teaching lessons as long as you are able to maintain some flexibility through a strategy such as a challenge table.

Another good idea is to consider letting students choose their own groups on the proviso that everyone is included and that everyone is working. Students often choose to work hard as long as they can work with their friends. Explain to the class that if anyone is not included or the students aren't working you will break up all of the groups and choose them yourself. This usually ensures good working and good behaviour.

I do have one more tip for managing group work: don't have groups bigger than three students to start with. When groups get bigger than three I usually find that someone is letting everyone else do the thinking. Pairs or threes usually work well.

Managing differentiation:

How to cater for students working at very different levels without driving yourself nuts

Setting the scene:

The reality of modern classroom teaching is that no teacher has a class of students who are all working at the one level. Teachers who aim their maths lesson at only one or two groups of learners are choosing to believe in a myth that this type of class still exists, if in fact it ever did. Within every single class, even if students are streamed, we have a diversity of learners to cater for. And for a single teacher with 25 or so different students to teach, this can be very difficult to handle.

This fact first hit home hard for me when I had a grade 5 class to teach. With 31 students in the class I struggled to keep my students with impairments and my extension students all actually learning. I quickly realised that running multiple activities within the one class was not practical unless at least some of the groups were just doing “busy work” and therefore not needing my help. Unfortunately that cut down the amount of “learning time” fairly significantly. There had to be a better way.

I decided to approach teaching maths in a bit of a back-to-front way and see how we went. Instead of getting started on teaching the next concept I asked questions orally to find out what my students already knew. I would often try to lead them down the garden path a little to work out if they really understood concepts, or if they just had the procedures memorised. Once I had worked out where their holes were, I would make up a problem that involved the next step in understanding, and leave them to try to solve it without me first explaining how to do it. For support students, I simply adjusted the content in the problem down until it was a bit beyond their grasp but not too far out of reach. For my extension students I took the same basic question and altered it to involve working backwards, multiple steps, filling a gap or a non-standard representation. Students worked in pairs or threes on their problems, trying multiple approaches and sharing their thoughts so far with the rest of the class. Together we analysed each idea and compared it to what we already knew of mathematical principles to check that the idea was sound.

This new approach led to some rather surprising discoveries. The first was the sheer volume of maths that students could actually work out for themselves when I just encouraged them to give it a go. After six months my traditional explanations virtually vanished from the class as my students learned to generalise their findings and express them as principles, algorithms or formulae. The second surprise was that many fundamental mathematical principles that I believed that the students had worked out years previously turned out to be pretty shaky, and some were missing altogether. The third, and perhaps most important discovery, was that many of my support students who had previously performed poorly at maths often improved so rapidly that most of them were performing at grade standard within 12 months.

As it turned out, many of my support students had been relying on memorisation without understanding underlying concepts. They also had a few fundamental misconceptions about basic principles. Using problem-based teaching allowed these to come out, and challenged students to work out whether or not their ideas were feasible. Once they self-corrected some misconceptions and started generalising about mathematical principles they suddenly started “getting it” and didn’t have to rely so heavily on memorisation.

Tips for getting differentiation to work:

A number of very simple strategies can make a big difference to how easy it is to differentiate for your students. My top tips include:

1. Start with one basic question and then adapt the content in the question down without changing the format. This provides support students with the same thinking challenge, but with content that is more appropriate. For example:
How many ways can you make the number 372 using hundreds, tens and ones?
 - Can you make it using hundreds, tens and ones blocks so that it is still 372?
 - How many ways can you make 72 using tens and ones?
 - How many ways can you make 23? Is 23 the same as 32? They both have the digits 2 and 3 so are they the same or different? How could you know?
 - How many ways can you make 6?
2. Start with one basic question and then think of “what if...” scenarios to add complexity for extension students. This increases the difficulty of the thinking skills significantly without necessarily increasing the content load. “What if” questions turn a standard problem into something far more difficult – such as by working backwards, filling a gap, creating more steps or adding complexity.
 - *What if* you could only use 1 hundred to make 372? How many ways could you make it now just using additional tens and ones?
 - *What if* you could only use tenths and hundredths?
 - *What if* you had to start with 1 hundred, 32 tens and 8 ones? What else could you put with it to make 372?
 - *What if* you could only use halves, thirds and quarters to make 372?
 - *What if* you could use three numbers, but two of them had to be the same? How could you make it now?
 - *What if* one of the two numbers had to be 139.7? What are your options for the other numbers?
3. Group for behaviour rather than for “ability” or “mixed ability”. Usually teachers feel quite anxious about how to choose groups rather than accepting the simple fact that we group according to which students will *not try to kill each other*. I often allow students to choose their own groups (maximum of 3 in a group), as long as no one is excluded and they are all working. If any group misbehaves then they are split up. It is quite a lot of fun to be able to use the threatened loss of maths privileges as a behaviour management tool!
4. Have a *Challenge Table*: Keep one spare table in your room with about four chairs at it. This allows you to have a very flexible space to cater for different students as they need it. I use this set up in a number of different scenarios:
 - a. Once I have set a problem: “If you understand what to do, you can go back and work with your partners now. If not, come to the challenge table and we’ll have a talk about it”. This stops *random wandering* by students who have no idea how to start.
 - b. Once we have come back together and shared ideas, and students have realised that their initial ideas were not ever going to work and they need to try something different: “If you want to change your mind you can go back and work with your partners now. If you think you are right, come to the challenge table.” This then leaves me with two distinct groups: the ones who are totally wrong and have no idea, and the ones who are right or pretty close to being right. I usually send the group who are right/almost right to the challenge table to work out who is right, or to complete a “what if...” question while I work with those who still have misconceptions.
 - c. When I cross-reference names with observations it often appears that I seem to consistently miss particular students. These are usually the quiet students who seem to blend into the background rather than grabbing for attention. When I see this happening, I invite those students to work with me at the challenge table for five minutes so that I can check on where they are up to.
 - d. I like to call students with similar communicating styles to work at the challenge table, such as all those who are quiet and just agree with everyone else in their group. When a group is formed with just those

students one of them eventually has to try something to solve the problem. Alternatively, try calling all the dominant students to fight it out amongst themselves. That frees up the other students to think for themselves.

5. Use *Tip Cards*, numbered and blue-tacked to the board, for a problem. The tips should increase in the amount of help that they offer to groups. A group of students who become stuck while working on a problem can decide to go and get one of the tips to give them a clue. They record the tip number on their books. They can then try to solve the problem again, or decide to get another tip. Usually they want to get as few tips as possible, so they work together pretty well to try and solve the problem without getting more cards.
For example (from *Back-to-Front Maths*, grade 3 Teaching Resource Book p47):
 - Is there a different combination of blocks that you could use so that you would still have 372? Can you make it without using 3 hundreds, 7 tens and 2 ones and still make it to be the same size?
 - How many hundreds do you have? If you only had 2 hundreds blocks, is there a way that you could use other blocks as well so that you could still make 372?
 - Let's just look at the 72. How many tens blocks are there? How many ones blocks are there? Is there another combination of blocks that you could use to make 72?
 - Make 6 tens and 12 ones on one side and 7 tens and 2 ones on another side. Are these amounts the same? How do you know? If you line them all up into a straight line will they be the same length? What does that tell you?
6. Use *Differentiated Problems* on coloured cards and allow students to work out which problem they want to try and solve. The base level problem goes in the middle, two above and two below are also present. Each student has to solve at least two levels of the problems. This way everyone can start on the base level problem, drop down if it is too hard, and go up if it is too easy. Once they solve one problem they automatically need to go up a level and try the harder question as well.

Once I started using problem-based teaching to differentiate it quickly becomes addictive. It is just so exciting to see students at all levels being challenged to think mathematically and actually work something out for themselves. If you are ready to have a go, I would recommend getting a hold of *Back-to-Front Maths* as it has inbuilt suggestions for how to adjust every single journal problem to suit learners at all levels. The series has been written with real teachers in mind, not those with classes that only exist in educational videos in which every student is working at the same level and nobody ever throws a chair!

Improving student reasoning:

Simple descriptions of what to look for when assessing Reasoning and Understanding as well as sample questions to ask and tips for helping students to improve

What is Reasoning in its simplest form?

The explanation below is my simplified version of what it means to Communicate or Reason. This is only meant to provide a starting point for discussion.

Reasoning involves the **demonstration of the mathematical process** that a student has gone through to solve a problem. This may include their working or equations, an oral explanation or set of questions and answers, a physical demonstration using materials, a diagram or model, or a written explanation etc. Mathematical communicating is about how clear and accurate the mathematical process is, not about literacy skills.

Some questions to consider when examining communicating include:

- What process have they used? Is this process mathematically valid? Have they shown enough of their process to be able to follow it?
- How well structured is their process? Is it logical and well-reasoned? Does it give enough detail? How much do I need to interpret or “read into” this process?

Useful questions for consideration when grading Reasoning:

1. Is the student’s process, as they have shown it, mathematically valid (will it generate the right answer every time)?
2. Is the student’s process relatively easy to follow, or does it require interpretation?
3. Is the student’s process detailed and logically structured such that someone exactly following the steps demonstrated or explained would get the same answer?

Reasoning Tips and Strategies:

Questions to elicit Reasoning:

- How did you get your answer/s?
- What operations did you use? With which numbers?
- What did you do first? Then what did you do after that?
- How do you know that you’re right? How do you know that the answer wasn’t ____ instead? Prove it to me.
- What do you mean in this part? Explain it to me.

Common problems and some useful tips:

1. When asked how he/she came up with their solution student answers, “I just knew it”.
 - Change the numbers in the question and ask them the steps for solving the new question.
 - Ask the student to tell you how to work out the answer to the new question without telling you the answer. “As soon as you tell me the answer then I won’t be able to mark it. I want to try and follow your steps and see if I can get the answer without you telling it to me.” Scribe for the student as needed.
2. Student had the answer “pop” into his/her head and has no idea how to prove it, or what thinking they went through to get there.
 - Ask the student to “help” someone who is stuck. Watch what they do and scribe their talking.
 - Deliberately make a mistake or give the wrong answer and have the student correct you. “So if I... does that work? What do you think? Prove it.”
 - Give the student other similar questions to solve and ask them to show you what is the same about each of them. “How is this question kind of the same as this other question? What is similar? What do you have to do to solve each of them?”

3. Student has trouble showing all of the steps and tends to skip bits.
 - Give the student sentence starters or equation starters with parts to complete. You might even provide words, phrases or numbers on sticky labels to stick into the spaces. Make sure that you provide lots of wrong ones as well so that they can't just complete the sentences with the only words that fit!
 - Have the student write the "calculator buttons to press" onto boxes. Show a calculator at the side so that students can only select from the numbers, operations and signs on the calculator.
 - Give the student new numbers and go through their process, skipping all of the bits that they skip. Give him/her a chance to correct you when you make mistakes and then go back and add those parts into his/her own process.

So... where to next?

If you are ready to give problem-based teaching a try, we recommend using *Back-to-Front Maths* for at least the first year. It is a highly-flexible resource that still provides a great deal of support for teachers. It is also recommended by the mathematical associations of South Australia and Western Australia.

Options:

There are many different options for how to use *Back-to-Front Maths* in your classroom. Below are a selection. Please contact Tierney Kennedy to discuss your individual needs and how we can help to meet them.

Option 1: Student workbooks and free web access

We recommend this option for most schools looking to implement problem-based teaching for the first time, or who are ready to implement the Australian Curriculum. This is also the option that has shown the most consistent improvement in student results.

Students are supplied with Thinking Journals (years 1-7) and Blasts Books (years 3-7) and teachers use the lesson plans in the Teaching Resource Books. This supplies a three-day program, allowing you two days per week for practicing skills, catching students up on missing content and conducting additional investigations.

Thinking Journals contain Problem Solving tasks, that help teachers uncover and deal with deeply held student misconceptions. Each problem is designed to be adapted to suit both support and extension students in a single lesson to cater for differentiation. Targeted questioning by the teacher directs students to evaluate and self-correct leading them to develop new Understanding. Assessment is built into every Journal Problem, with simple clear criteria.

Blasts Books develop Fluency and Understanding by logical, leading questions that focus on mathematical patterns and underlying principles. They also contain challenging non-standard problems of a similar fashion to NAPLAN questions.

Teaching Resource Books Explicitly set out problem-based pedagogy focusing on questioning, misconceptions and differentiation.

In addition to the student workbooks, web access for all teachers for one year is supplied free. This enables teachers to download and photocopy any lesson from any grade for use with their students, as well as providing additional moderating and assessment tasks, investigations, and planning tools.

Option 2: Teaching Resource Packs and paid web access

For schools who are already familiar with problem-based teaching and are looking to move away from student workbooks we recommend using a combination of Teaching Resource Packs and web access. Schools who have begun by just using web-access have mostly had improved student results, but not as consistently as those who choose the workbook option initially.

Teaching Resource Packs for each grade contain one copy of the Teaching Resource Book, Thinking Journal and Blast Book for that grade. These books are not photocopyable. School site-licence subscriptions are available to www.backtofrontmaths.com.au/teachers for 12 month periods, allowing access to every lesson, assessment task, investigation and planning tool and limitless photocopying for the licence period.

Option 3: Books and program writing

A number of schools have opted to have our consultants work with their teachers to create a school-based work program. This includes a number of single or double teacher schools with multi-age classes. This individual program is then supported by Thinking Journals and Blasts books for the students, and by the inclusion of free web-access.

Option 4: Teaching Resource Packs or Individual web access

For individual teachers who wish to implement problem-based teaching we recommend first trialling a Teaching Resource Pack or Individual web subscription. This will allow an individual to try out teaching ideas before committing to whole-school change.

Once you have decided what you think you would like to use as a school, please contact Tierney Kennedy at Kennedy Press or your local branch of AAMT to determine how to best proceed.

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